

Direct Instantons and Nucleon Magnetic Moments*

M. Aw[†] and M.K. Banerjee

Department of Physics, University of Maryland, College Park, MD 20742-4111

H. Forkel

Institut für Theoretische Physik, Universität Heidelberg, D-69120 Heidelberg, Germany

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Abstract

We calculate the leading direct-instanton contributions to the operator product expansion of the nucleon correlator in a magnetic background field and set up improved QCD sum rules for the nucleon magnetic moments. Remarkably, the instanton contributions are found to affect only those sum rules which had previously been considered unstable. The new sum rules show good stability and reproduce the experimental values of the nucleon magnetic moments with values of χ , the quark condensate magnetic susceptibility, consistent with other estimates.

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[†]Present address: Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213.

E-mail: mountaga@cmu.edu

For over two decades QCD instantons have been associated with fundamental aspects of strong interaction physics, as, for example, with the θ -vacuum [1], the issue of strong CP violation [1], and the anomalously large η' mass [2]. Unequivocal and quantitative evidence for their role in hadron structure, however, turned out to be much harder to establish. This is mainly due to the complexity involved in dealing with interacting instanton ensembles and their coupling to other vacuum fields over large distances.

Instanton vacuum models [3] attack the first part of this problem directly, by approximating the field content of the vacuum as a superposition of solely instantons and anti-instantons. This approach has been developed for more than a decade and can describe an impressive amount of hadron phenomenology [3]. More recently, QCD lattice simulations began to complement such vacuum models by isolating instantons in equilibrated lattice configurations and by studying their size distribution and their impact on hadron correlators [4]. While the results obtained with different, currently developed lattice techniques have not yet reached quantitative agreement, they do confirm the overall importance of instantons and some bulk properties of their distribution in the vacuum.

Another approach towards linking the instanton component of the vacuum to hadron properties has been developed over the last years by generalizing the nonperturbative operator product expansion (OPE) and QCD sum rule techniques [5–7]. While its range of applicability is more limited than that of instanton vacuum models and of lattice calculations, it avoids the need for large-scale computer simulations and takes, in contrast to instanton models, all long-wavelength vacuum fields and also perturbative fluctuations into account. Furthermore, the approach is largely model-independent and allows the study of instanton effects in a fully analytic and, therefore, rather transparent fashion.

In the present paper, we adapt this approach to properties which characterize the response of the hadronic system to a weak external field. Specifically, we calculate the direct-instanton contributions to the nucleon correlator in the presence of a constant electromagnetic field. With the help of background-field sum-rule techniques due to Ioffe and Smilga [8] and Balitsky and Yung [9], we then study previously neglected instanton effects in the

QCD sum rules for the magnetic moments of the nucleon.

As already mentioned, our work is based on the nucleon correlation function

$$i \int d^4x e^{ipx} \langle 0 | T \eta(x) \bar{\eta}(0) | 0 \rangle_F = \Pi_0(p) + \sqrt{4\pi\alpha} \Pi_{\mu\nu}(p) F^{\mu\nu}, \quad (1)$$

in the background of a constant electromagnetic field $F_{\mu\nu}$. The interpolating fields $\eta(x)$ with proton or neutron quantum numbers [10] are composite operators of massless up and down quark fields:

$$\eta_p(x) = [u^{aT}(x) C \gamma_\alpha u^b(x)] \gamma_5 \gamma^\alpha d^c(x) \epsilon^{abc}, \quad \eta_n = \eta_p(u \leftrightarrow d). \quad (2)$$

For the application in QCD sum rules we need a theoretical description of the correlator (1) at momenta $s = -p^2 \simeq 1\text{GeV}^2$, i.e. at distances $x \lesssim 0.2\text{fm}$.

The information on the magnetic moments is contained in the second term on the right-hand side of Eq. (1). It characterizes the linear response of the nucleon to the external field and can be decomposed into three independent Lorentz and spinor structures:

$$\Pi_{\mu\nu}(p) = (\not{p} \sigma_{\mu\nu} + \sigma_{\mu\nu} \not{p}) \Pi_1(p^2) + i(\gamma_\mu p_\nu - \gamma_\nu p_\mu) \not{p} \Pi_2(p^2) + \sigma_{\mu\nu} \Pi_3(p^2). \quad (3)$$

Note that the invariant amplitude Π_1 corresponds to the chirally-even part of the correlator, while Π_2 and Π_3 are associated with the chirally-odd part.

The nonperturbative OPE [11,8] of the above correlator at small distances can be generated by splitting each diagram contributing to (3) in all possible ways into a hard and a soft subgraph. The hard subgraphs contribute to the Wilson coefficients and are usually calculated perturbatively, with the integration range of each internal momentum restricted¹ to be larger than the OPE scale $\mu \sim 0.5\text{GeV}$. The soft subgraphs correspond to hadron-channel independent condensates, renormalized at μ . In the presence of an external electromagnetic field the OPE (up to eight-dimensional operators) involves the additional, Lorentz-covariant condensates

¹In practice, this restriction is often unnecessary (see below).

$$\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle = \sqrt{4\pi\alpha}\chi F_{\mu\nu}\langle 0|\bar{q}q|0\rangle, \quad (4)$$

$$g\langle 0|\bar{q}G_{\mu\nu}q|0\rangle = \sqrt{4\pi\alpha}\kappa F_{\mu\nu}\langle 0|\bar{q}q|0\rangle, \quad (5)$$

$$g\langle 0|\bar{q}\gamma_5\tilde{G}_{\mu\nu}q|0\rangle = \frac{i}{2}\sqrt{4\pi\alpha}\xi F_{\mu\nu}\langle 0|\bar{q}q|0\rangle. \quad (6)$$

($G_{\mu\nu} = \frac{1}{2}\lambda_a G_{\mu\nu}^a$, $\tilde{G}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}G_{\rho\sigma}$ with $\varepsilon_{0123} = -1$.) The parameters χ , κ , and ξ play the role of generalized susceptibilities and quantify the vacuum response to weak electromagnetic fields. The magnetic susceptibility of the quark condensate, χ , for example, originates from the induced spin alignment of quark-antiquark pairs in the vacuum. Note also that χ is associated with the lowest-dimensional induced condensate, which enhances its role in the OPE and the corresponding sum rules.

The OPE of $\Pi_{\mu\nu}(p)$ up to operators of dimension eight, with perturbatively calculated Wilson coefficients, has been obtained in Ref. [8]. An inherent assumption of this calculation - and of the QCD sum rule program in general - is that the short-distance physics associated with fields of wavelength smaller than μ^{-1} is predominantly perturbative. It is well known, however, that also strong nonperturbative fields of rather small size exist in the QCD vacuum. Instantons, i.e. the finite-action solutions of the classical, Euclidean Yang-Mills equation [12], are paradigmatic examples of such fields and have a crucial impact on the vacuum structure.

The nonperturbative contributions from short-wavelength fluctuations of quarks and gluons around instantons are thus neglected in the standard treatment of the OPE. Their relative importance, and hence the justification for approximately disregarding them, depends both on the instanton size distribution in the vacuum and on the quantum numbers of the hadronic channel under consideration. Instantons of smaller (average) size $\bar{\rho}$ are accompanied by fluctuations of smaller wavelength, and those contribute more strongly to the Wilson coefficients. The hadron-channel dependence of the instanton contributions originates mainly from the chirality and spin-color coupling of the quark zero-modes in the instanton background.

Instanton-induced effects are particularly large in the pseudoscalar-isovector and scalar-

isoscalar channels, because there the quark zero-modes contribute with maximal strength. As a consequence, instanton contributions dominate already at short distances in the pseudoscalar sum rules, and are essential for their stability [6]. In the vector and axial-vector channels, on the other hand, zero-mode contributions are, to leading order in the instanton density, absent.

The strength of instanton contributions to the nucleon channel lies about halfway in-between these extreme cases. In the chirally-odd amplitudes, it was found to be roughly of the same magnitude as that of the condensate contributions [5,7], since the spin-0 diquark operators in the interpolating fields (2) couple strongly to instantons. The nucleon channel is therefore well suited for studying the interplay between instantons and other vacuum fields [7].

The calculation of the leading direct-instanton contributions to the background-field correlator (1) proceeds essentially along the lines described in Refs. [5–7], to which we refer for more details. Similar to the condensates, the bulk properties of the instanton size distribution are generated by long-distance vacuum dynamics and have thus to be taken as input for this calculation. As before [5–7], we will use the standard values [13] $\bar{\rho} \simeq \frac{1}{3} \text{ fm}$ for the average instanton size and $\bar{R} \simeq 1 \text{ fm}$ for the average separation between neighboring (anti)instantons. The results of instanton vacuum models [3,14] confirm these scales, while those from the lattice [4] are not yet fully consistent but lie in the same range (with maximal deviations of about 50 %).

Since the average instanton size is of the order of the inverse OPE scale, $\bar{\rho} < \mu^{-1} \simeq 0.4 \text{ fm}$, instanton corrections to the Wilson coefficients can be substantial. Moreover, since $\bar{\rho}^{-1} \gg \Lambda_{QCD}$, these corrections are essentially semiclassical, and at the relevant distances $x \lesssim 0.2 \text{ fm} \ll \bar{R}$ multi-instanton correlations should be negligible. The instanton contributions to the OPE coefficients can therefore be calculated in semiclassical approximation, i.e. by evaluating the correlator (1) in the background of the instanton and anti-instanton field and by then averaging the instanton parameters over their vacuum distributions. (The distribution of the instanton’s position and color orientation is uniform, due to translational

and gauge invariance).

We treat the zero-mode sector of the quark propagator in the instanton field exactly and approximate the continuum modes, as before [5], by plane waves. The recently found zero-mode dominance of the ground-state contributions to the pion and ρ -meson correlators on the lattice [15] supports the validity of this approximation. The impact of the remaining vacuum fields (including other instantons) on the instanton contributions is accounted for in a mean-field sense [16] and generates an effective mass $\bar{m}(\rho) = -\frac{2}{3}\pi^2\rho^2\langle\bar{q}q\rangle$ for the quark zero modes. We further use the approximate instanton size distribution $n(\rho) = \bar{n}\delta(\rho-\bar{\rho})$ [13] and the self-consistency condition [17] $\langle\bar{q}q\rangle = -2\int d\rho\frac{n(\rho)}{\bar{m}(\rho)} = -2\frac{\bar{n}}{\bar{m}(\bar{\rho})}$ (which is numerically satisfied to good accuracy) to eliminate the \bar{n} dependence from the resulting expressions.

Up to operators of dimension eight, we find the leading instanton contributions to the correlator (1) to arise from just one type of graph, in which two of the quarks (emitted from the current (2) at $x = 0$) propagate in zero-modes while the third interacts with the background field through the magnetized quark condensate. Graphs in which the background field couples directly to a hard quark in a zero-mode, vanish. The same holds for graphs in which the interaction with the photon causes a transition from zero-mode to continuum-mode propagation². The contribution from direct instantons is thus generated by the interplay between the rather localized quark zero modes and more slowly varying, nonperturbative vacuum fields. Effects of this type appear naturally in the OPE, whereas they are difficult to account for in, e.g., quark models [7].

Evaluating the corresponding graphs in the instanton and magnetic background fields as described above, and averaging over the instanton size distribution, we arrive at

$$\langle 0|T\eta_p(x)\bar{\eta}_p(0)|0\rangle_{F,inst} = \Pi_0^{inst}(x) - \frac{2^3 e_u}{3\pi^4} \frac{\bar{\rho}^4}{\bar{m}^2} \langle \bar{q}\sigma_{\mu\nu}q \rangle_F \sigma_{\mu\nu} \int d^4x_0 \frac{1}{(r^2 + \bar{\rho}^2)^3 (x_0^2 + \bar{\rho}^2)^3} \quad (7)$$

($r = x - x_0$, where x_0 specifies the center of the instanton) for the proton. The corresponding

²Contributions of this type are essential in the pseudoscalar three-point correlator associated with the pion electromagnetic form factor [6].

neutron correlator is obtained by replacing e_u with e_d . In order to put the instanton contribution (7) to use in the sum rules, we also need its Fourier and Borel transform³. Again for the proton, it reads

$$\hat{\Pi}_3(M^2) = \frac{e_u}{128\pi^4} a \chi \bar{\rho}^2 M^6 I(z^2). \quad (8)$$

Here, M denotes the Borel mass parameter. We have also used the standard definitions $z \equiv M\bar{\rho}$, $a \equiv -(2\pi)^2 \langle \bar{q}q \rangle$, and abbreviated the integral

$$I(z^2) = \int_0^1 \frac{d\alpha}{\alpha^2(1-\alpha)^2} e^{-\frac{z^2}{4\alpha(1-\alpha)}} = 4e^{-\frac{z^2}{2}} \left[K_0\left(\frac{z^2}{2}\right) + K_1\left(\frac{z^2}{2}\right) \right]. \quad (9)$$

Note that (9) shows the typical exponential Borel-mass dependence of instanton contributions [5]. Together with the appearance of the new scale $\bar{\rho}$, this distinguishes them from the logarithms and power terms of the standard OPE.

An important qualitative property of the instanton contribution has been made explicit in Eq. (8): to leading order, direct instantons contribute almost exclusively⁴ to one invariant amplitude, Π_3 , which is associated with the chirally-odd Dirac structure $\sigma_{\mu\nu}$. Exactly this amplitude was singled out in the previous sum rule analysis of Ref. [8], for two reasons: first, one of its Wilson coefficients contains, in the “pragmatic” version of the OPE (see below), an infrared divergence. Secondly, the Π_3 sum rule failed to show a fiducial Borel region of stability [18] (even after proper subtraction of the divergence), while the other two sum rules provide stable and accurate values for the nucleon magnetic moments even without direct-instanton corrections, and with a value of χ consistent with other, independent estimates [19].

³For the Borel transform we follow the convention of Ref. [10].

⁴There is also a small direct-instanton contribution of similar structure to the amplitude Π_2 . This term turns out to be too small to have an appreciable impact on the corresponding sum rules, however, and will not be discussed further.

To understand the first point we note that, since in QCD perturbative contributions from soft loop momenta are normally small compared to the corresponding condensate contributions, it is standard procedure not to remove them in sum-rule calculations [20]. This simplification - which goes under the name of “pragmatic OPE” - fails, however, if infrared divergences appear in diagrams associated with a Wilson coefficient. Such an infrared divergence was encountered in the OPE of Π_3 , in a graph where a vacuum gluon field and the background photon interact with the same hard quark line. Hence the contribution from soft loop momenta has to be cut off explicitly, according to the rules of the exact OPE. A similar divergence was found before [21] in the vector meson correlator when two soft gluon fields couple to the same quark line.

The authors of [8] conjectured that the appearance of infrared singularities in Π_3 and the absence of a stability region in the associated sum rule might be related. This seems unlikely, however, in view of the later finding [22] of similar infrared divergencies (which always occur when a quark line interacts with multiple soft gauge fields) in the other two amplitudes of (3), which nevertheless lead to satisfactory sum rules. Direct instantons, on the other hand, contribute almost exclusively to Π_3 , and their previous neglect could offer a more plausible explanation for the instability of the Π_3 sum rule. Support for this conjecture, which we are going to test quantitatively below, comes from two other chirally-odd nucleon sum rules (for the nucleon mass [5] and its isospin splitting [7]), where exactly such a selective stabilization due to instantons has been found.

Our modified Π_3 sum rule is obtained by equating the Borel transform of the standard OPE from Ref. [8] and the instanton contributions (8) to the Borel-transformed double dispersion relation for the correlator (1), with a spectral function parametrized in terms of the nucleon pole contribution (containing the magnetic moments) and a continuum based on local duality. Including the infrared-divergent term encountered in Ref. [8], duly truncated

at the OPE renormalization point, the new sum rule for the proton reads⁵

$$aM^2 \left\{ \left[e_u - \frac{1}{6}e_d(1 + 4\kappa + 2\xi) \right] E_1(M) + \frac{1}{6}e_u \frac{m_0^2}{M^2} \left[\ln \frac{M^2}{\mu^2} - \gamma_{EM} \right] L^{-\frac{4}{9}} \right. \\ \left. + \frac{1}{6}e_d M^2 \chi E_2(M) L^{-\frac{16}{27}} - \frac{1}{8}e_u \chi \rho_c^2 M^4 I(z^2) L^{-\frac{16}{27}} \right\} = \frac{1}{4} \tilde{\lambda}_N^2 m e^{-\frac{m^2}{M^2}} \left[\frac{\mu_p}{M^2} - \frac{\mu_p^a}{2m^2} + A_p \right], \quad (10)$$

where m is the nucleon mass, W the continuum threshold, and λ_N the coupling of the current (2) to the nucleon state, $\langle 0|\eta|N\rangle = \lambda_N u$. For the mixed quark condensate we use the standard parametrization $\langle \bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle = -m_0^2\langle \bar{q}q\rangle$ with $m_0^2 = 0.8 \text{ GeV}^2$, and $\gamma_{EM} \simeq 0.577$ is the Euler-Mascheroni constant. The additional parameters $A_{p,n}$ determine the strength of electromagnetically induced transitions between the nucleon and its excited states. The sum rule for the neutron is obtained from (10) by interchanging e_u and e_d and by replacing $\mu_p, \mu_p^a \rightarrow \mu_n$ and $A_p \rightarrow A_n$. We have also defined $\tilde{\lambda}_N^2 = 32\pi^4\lambda_N^2$, $L = \ln(M/\Lambda)/\ln(\mu/\Lambda)$ ($\Lambda = 0.1 \text{ GeV}$), and transferred, using the standard expressions

$$E_n(M) = 1 - e^{-\frac{W^2}{M^2}} \left[1 + \sum_{j=1}^n \frac{1}{j!} \left(\frac{W^2}{M^2} \right)^j \right], \quad (11)$$

the continuum contributions to the OPE-side of the sum rules. The appropriate form of these contributions has recently been clarified in Ref. [23].

In principle, several alternative options are available for the quantitative analysis of background-field sum rules. In practice, however, one is limited by the fact that their fiducial domain (i.e. the Borel-mass region in which the neglect of higher-order terms in the short-distance expansion is justified while the nucleon pole still dominates over the continuum) is generally not large enough to determine all the unknown parameters from a stable fit. The authors of Ref. [8] succeeded, however, in eliminating the susceptibilities and other constants by combining the two sum rules for Π_1 and Π_2 and their M^2 -derivatives. The magnetic moments can then be fitted and are in good agreement with experiment, although taking derivatives of the sum rules generally reduces their reliability. Unfortunately, the

⁵We have corrected some errors which appeared in the expressions of Ref. [8].

above procedure also eliminates the continuum contributions, which makes it impossible to determine whether a fiducial stability domain exists.

In any case, this procedure is ineffective for our new sum rules (10), as it cannot eliminate the additional χ -dependence introduced by the direct-instanton contribution. Similarly, working with the ratio of background-field and mass sum rules, as done in Ref. [24] for the Π_1 sum rules, offers no advantage in our case since already the leading terms in the OPE of the chirally-odd mass and background-field sum rules differ. Therefore, we resort to a direct minimization of the relative deviations between the two sides of Eq. (10). The coupling $\tilde{\lambda}^2 = 2.93 \text{ GeV}^6$ and the continuum threshold $W = 1.66 \text{ GeV}$ are, following the procedure of Ref. [8], obtained by fitting the instanton-improved nucleon mass sum rule [5] to the experimental nucleon mass⁶. The values of the two susceptibilities $\kappa = -0.34 \pm 0.1$ and $\xi = -0.74 \pm 0.2$ were estimated in independent work by Kogan and Wyler [25]. This enables us to fit both sides of the sum rules (10) by varying χ and A_p (or A_n , respectively) while keeping the magnetic moments fixed at their experimental values.

The fits are performed in the fiducial Borel mass domain, which is bounded from below by requiring the highest-dimensional operators to contribute at most 10 % to the OPE and from above by restricting the continuum contribution to maximally 50 %. The resulting fiducial domains of both the proton and neutron sum rules, $0.8 \text{ GeV} \lesssim M \lesssim 1.15 \text{ GeV}$, are larger than those of the sum rules based on Π_1 and Π_2 .

The fit results are shown in Fig. 1, where for both the proton and the neutron sum rules the direct-instanton contributions, the remaining OPE including the continuum contributions, their sum (which makes up the left-hand side of Eq. (10)), and the right-hand sides are plotted. The fit quality of both the proton and neutron sum rules is excellent. As previously in the instanton-improved, chirally-odd nucleon mass sum rule [5], the theoretical

⁶We have included anomalous-dimension corrections in the mass sum rule of Ref. [5] and restored the full four-quark condensates, since we are dealing exclusively with Ioffe's interpolating field (2).

side of the sum rules, including the instanton-induced part, is almost indistinguishable from the phenomenological side. Moreover, Fig. 1 shows that the direct-instanton contributions can reach about half the magnitude of the remaining terms in the OPE, which makes it evident why their previous neglect had a detrimental impact on the stability properties.

An alternative way of evaluating the optimized sum rules consists in solving them for μ_N and plotting the result as a function of the Borel mass, as shown in Fig. 2. The resulting functions $\mu_{p,n}(M)$ specify the value of the magnetic moment which is required to make both sides of the sum rule (10) match exactly at each value of the Borel mass. The instanton-corrected sum rules render $\mu(M)$ practically M -independent, thereby again indicating an almost perfect fit for both proton and neutron.

This fit predicts $\chi \simeq -4.96 \text{ GeV}^{-2}$ for the proton and $\chi \simeq -4.73 \text{ GeV}^{-2}$ for the neutron sum rule. These values correspond to the OPE scale $\mu = 0.5 \text{ GeV}$ adopted for our sum rules and lie inside the range obtained from other estimates [8,19,22]. They are somewhat smaller in magnitude than the value $\chi \simeq -5.7 \text{ GeV}^{-2}$ found in the two- and three-pole models of Ref. [19]. (Our predicted values for the excited-state transition parameters are $A_p \simeq 0.28 \text{ GeV}^2$ and $A_n \simeq -0.27 \text{ GeV}^2$.)

In conclusion, we have recovered a third reliable sum rule for the nucleon magnetic moments. In contrast to the other two, it receives previously neglected direct-instanton contributions which arise from the interplay with long-wavelength vacuum fields. Our new sum rule is built on the chirally-odd amplitude Π_3 of the nucleon correlator in an electromagnetic background field and found to be at least as stable as the other two, although it had previously been regarded as flawed. The new sum rule adds to the predictive power of the background-field sum rules and strengthens their mutual consistency.

Furthermore, our results reinforce a systematic pattern which emerged from previous studies of direct-instanton effects both in the nucleon and pion channels: those sum rules which worked satisfactorily without instanton corrections receive little or no direct instanton contributions, and previously less reliable or completely unstable sum rules are stabilized by large instanton contributions. This pattern points not only towards the importance of

direct instantons in particular sum rules, but also supports the adequacy of their semiclassical implementation into the OPE. Our results show that these conclusions continue to hold in the presence of a “magnetized” vacuum.

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FIGURES

FIG. 1. The OPE (dashed line) and direct instanton (dotted line) contributions to the new $\sigma_{\mu\nu}$ sum rules for the proton (positive range) and neutron. Their sum (dot-dashed line) is compared to the RHS (solid line).

FIG. 2. The Borel mass dependence of the magnetic moments of the proton (upper) and neutron calculated from the optimal fit of LHS and RHS.



